Analysis of Call Blocking Probability in TDM/WDM Networks with Transparency Constraint

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***Abstract*—In this letter we devise and validate by simulation an analytical model to study the performance of TDM/WDM networks using a three-stage switching architecture as an abstract model. We consider the transparency property as a constraint: a new incoming call can be accepted only without modifying the routing of previously accepted calls. We concentrate on the analysis of the call blocking probability by varying the traffic pattern and the configuration of the switching architecture. We show that a very good agreement is obtained between simulation and analytical results.**

***Index Terms*—Modeling, switches.**

I. Introduction

WE BASE our model of a TDM/WDM network on a classical three-stage switching architecture, where each module (a switching matrix) in the first and third stage is linked by a dedicated channel with all modules in the second stage. The first and third stages include the same number N of modules, whereas M denotes the number of modules in the second stage and n the number of inlets (and outlets) in each first (third) stage module.

The three-stage architecture is an abstract model which can be considered as representative of many architectures. In [1] it has been shown that the time slot assignment (TSA) problem in a TDM system can be solved using well-known permutation-routing algorithms in three-stage rearrangeable nonblocking Clos networks, where $M = n$ represents the frame size in slots, and N is the number of input channels to the TDM switch.

As another example, consider the nationwide time-slotted WDM optical network developed in the framework of the ACTS Sonata project [2]. The network comprises 20 000 000 access nodes with wavelength agile transmitters and receivers. Nodes are partitioned among 400 passive optical networks (PONs) which are directly connected to a single passive wavelength routing node (PWRN), which creates a fully meshed network among PONs. At the PWRN, 400 wavelength converter arrays are available, each array comprising 400 wavelength converters. This scheme provides a rearrangeable wavelength routed network among PONs, that can be represented as a three-stage architecture, where each module represents a wavelength conversion/selection process. Since the first (and third) stage modules represent the PONs, then $N = 400$. Each second stage module represents an array of wavelength converters; thus, also $M = n = 400$. A centralized network controller allocates wavelengths and time slots so as to avoid conflicts. The wavelength and slot selection process is analogous to the route selection process in three-stage switching architectures.

A three-stage architecture is known to be strictly nonblocking by Clos theorem [3] if $M \geq 2n - 1$. The switch is rearrangeable nonblocking by Slepian–Duguid theorem when $M \geq n$. Each idle inlet can be connected to any idle outlet by applying, if necessary, a suitable rearrangement of already established calls. Paull theorem [3] asserts that the maximum number of connections to be rearranged is equal to $N-1$. Moreover, Paull theorem proof gives an $O(N^2)$ algorithm to reroute the calls.

However, in the context of high-speed switching networks, with very large N and n, both the time to execute the Paull algorithm and, even more significantly, the signalling bandwidth required to notify the modified routing of connections, may be excessive. For this reason, it is of interest to evaluate the call blocking probability when no rearrangement is performed, i.e., a call is accepted only if it can be routed through the switching matrix without rearranging previously established calls. We refer to this requirement as transparency constraint.

To determine performance indices in terms of blocking probabilities, we devise a general and accurate analytical model and develop a simulation tool to validate analytical results. The model is general since it is able to deal with mixed traffic patterns and it is accurate since a very good matching between analytical and simulation results is achieved in different scenarios. On the contrary, previously presented models, such as those proposed by Lee and Jacobaeus (see [3]), deal only with one type of traffic and provide results which are accurate only for low input traffic loads.

II. Problem Formulation

We denote by N the number of switching modules in the first and third stage of the switching matrix, whereas M denotes the number of switching modules in the second stage. Each module in the first (third) stage is linked by a dedicated channel with each module in the second stage. We use n to identify the number of inlets in each module in the first stage and the number of outlets in each module in the third stage. We use indices $i = 1, \cdots, N$ and $o = 1, \cdots, N$ to identify respectively a generic module in the first and third stage. Overall,

Manuscript received May 28, 1999. The associate editor coordinating the review of this letter and approving it for publication was Prof. V. S. Frost. This work was supported in part by the Italian Ministry for University and Research, by the National Research Council, and by European ACTS project AC351 SONATA

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Publisher Item Identifier S 1089-7798(00)02942-2.

the switching matrix architecture can be viewed as a three-stage switch with $N \times n$ input ports and $N \times n$ output ports.

We are interested in analyzing the performance of this architecture in terms of blocking probability. We impose the transparency constraint in our study: no reconfiguration of routes already set through the switch is allowed.

We consider a normalized traffic pattern described via a matrix P, where its element p_{io} represents the normalized amount of traffic from input module i to output module o; we assume that $\sum_{i=1}^{N}\sum_{o=1}^{N} p_{io} = 1$. If we envision a uniform traffic pattern, then $p_{io} = 1/N^2$. In the description of the model we will concentrate on a specific first stage module denoted as input module I and on a specific third stage module named output module O in the sequel.

We describe the call arrival and departure processes at input module I with a birth–death Markov chain MC_I, first assuming a Poisson process for call arrivals and negative exponential distributed call holding time. Later, we will extend the model to deal with batch arrivals. The Markov chain state k represents the number of active calls at input module I; obviously, $k \leq n$.

We denote by $P\{\text{block}(O)|k\}$ the call blocking probability in state k at input module I for calls directed toward output module O, i.e., the probability that a new call directed toward output O is blocked at input I, given that k calls are already active at input I, due to the unavailability of an idle path through the switching matrix. $P\{\text{inadm}(O)\}$ represents the probability that a call at input module I directed toward output module O is not admissible, i.e., the probability that a new call directed toward output O is discarded due to the unavailability of an idle outlet at output module O.

We denote by λ the mean call arrival rate, by $1/\mu$ the mean call holding time and by π_k^I the steady-state probabilities at input module I. In MC_I, the transition probabilities from state k to state $k-1$ are given by $k\mu$, whereas the transition probabilities from state k to state $k+1$ are given by

$$\lambda \sum_{o=1}^{N} (1 - P\{\text{block}(o)|k\})(1 - P\{\text{inadm}(o)\}) \frac{p_{Io}}{N}. \quad (1)$$

Similarly, we describe the call arrival and departure processes at output module O with a birth–death Markov chain MC_O, where state h represents the number of active calls in output module O and π_h^O the steady-state probabilities at output module O.

We now focus on the analysis of MC_I at input module I: we examine $P\{\text{inadm}(O)\}$ and $P\{\text{block}(O)|k\}$, i.e., the call inadmissible and blocking probabilities toward output module O.

We know that

$$P\{\text{inadm}(O)\} = \pi_n^O$$

For the computation of $P\{\text{block}(O)|k\}$, by the total probability theorem, we have

$$P\{\text{block}(O)|k\} = \sum_l \sum_h P\{\text{block}(O)|k,\, h,\, l\} P\{h,\, l|k\}$$

where the conditional probability $P\{\text{block}(O)|k,\, h,\, l\}$ represents the probability that a call is blocked given that k calls are active on input module I, h calls are active on output module O and l calls are directed from input module I to output module O.

By using the definition of conditional probability

$$\begin{aligned} &P\{\text{block}(O)|k\} \\ &\quad = \sum_l \sum_h P\{\text{block}(O)|k,\, h,\, l\} P\{l|h,\, k\} P\{h|k\}. \end{aligned}$$

By assuming that h and k are statistically independent, we obtain

$$P\{\text{block}(O)|k\} = \sum_l \sum_h P\{\text{block}(O)|k,\, h,\, l\} P\{l|h,\, k\} \pi_h^O.$$

Let us define $q = \min(h, k)$; obviously, $l \leq q$. We can approximate $P\{l|h,\, k\}$ by assuming Bernoulli distribution, so that

$$P\{l|h,\, k\} = \binom{q}{l} p_{IO}^l (1 - p_{IO})^{(q-l)}.$$

Finally, we have

$$\begin{aligned} &P\{\text{block}(O)|k\} = \\ &\quad \sum_l \sum_h P\{\text{block}(O)|k,\, h,\, l\} \binom{q}{l} p_{IO}^l (1 - p_{IO})^{(q-l)} \pi_h^O. \end{aligned} \quad (2)$$

In (2), p_{IO} is known and determined by the traffic pattern and the state probability π_h^O is obtained by solving the Markov chain. By enforcing that $l \leq k$ and $l \leq h$, a simple combinatorial computation gives

$$P\{\text{block}(O)|k,\, h,\, l\} = \frac{\binom{h-l}{M-k}}{\binom{M-l}{M-k}}.$$

A similar equation holds for $P\{\text{block}(I)|h\}$ at output module O:

$$\begin{aligned} &P\{\text{block}(I)|h\} = \\ &\quad \sum_l \sum_k P\{\text{block}(I)|h,\, k,\, l\} \binom{q}{l} p_{IO}^l (1 - p_{IO})^{(q-l)} \pi_k^I. \end{aligned} \quad (3)$$

The whole model comprises N MC_i, one for each input module i, and N MC_o, one for each output module o.

The blocking and inadmissible probabilities are obtained through an iterative process. We first set the values of $P\{\text{block}(o)|k\}$, and $P\{\text{inadm}(o)\}$ in the MC_i at each input module i and $P\{\text{block}(i)|h\}$ and $P\{\text{inadm}(i)\}$ in the MC_o at each output module o. Then, solving the Markov chains, we obtain π_k^i and π_h^o. Finally, using equations similar to (2) and (3), we compute the new values of $P\{\text{block}(o)|k\}$ (and $P\{\text{block}(i)|h\}$), that allow the computation of the new transition probabilities in (1). Now, a new iteration can start; we stop the process when the values of blocking and inadmissible probabilities at iteration i are comprised within a relative interval of 10^{-5} of the values at iteration $i-1$.

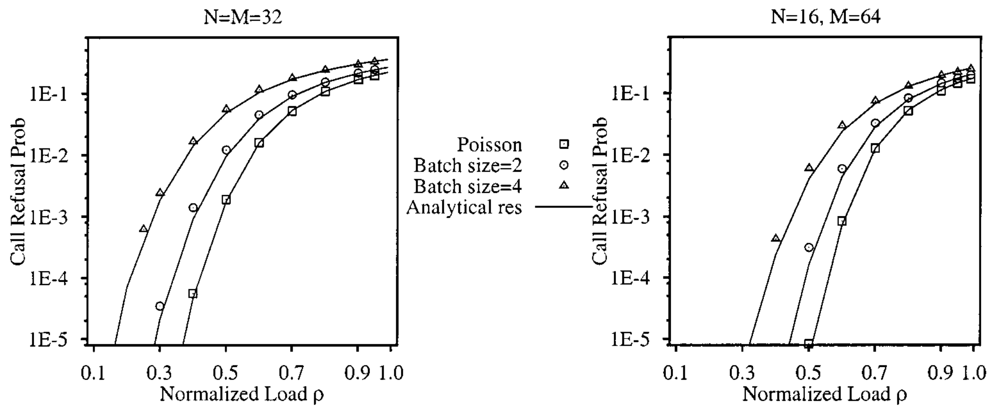

Fig. 1. Call refusal probability for uniform traffic pattern.

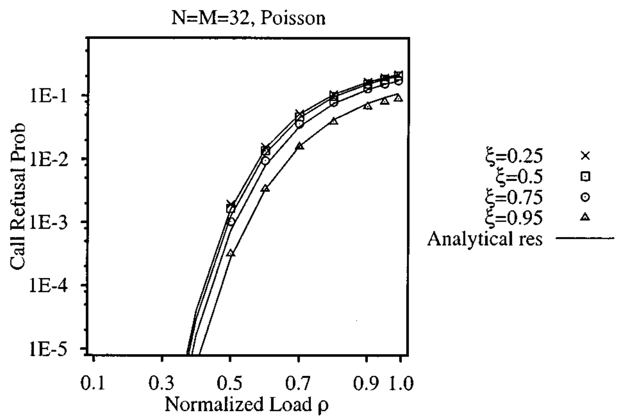

Fig. 2. Call refusal probability for unbalanced traffic pattern.

Observe that the model can be simply extended to deal with batch call arrivals instead of Poisson arrivals. The Markov chains are slightly more complex than birth–death Markov chains, but the iterative process remains the same. Batch arrivals increase the call burstiness, providing a more realistic traffic pattern. In Section III we present results for batch arrivals, with batch size of 1 (Poisson), 2, and 4 calls.

III. Performance Results

We compare performance results of the analytical model with simulation results, in order to assess the model effectiveness. All the simulation results were obtained by stopping the simulation runs when a 5% confidence interval with a 95% confidence level was achieved.

In Fig. 1 we plot the call refusal probability, i.e., the sum of the blocking and inadmissible probabilities, as a function of normalized load $\rho = \lambda/\mu$, taking the call batch size as a parameter, under uniform traffic pattern. On the left hand we consider a switching architecture with $M = n = N = 32$, whereas on the right hand we set $M = n = 64$, $N = 16$.

In Fig. 2 we plot the call refusal probability as a function of normalized load $\rho = \lambda/\mu$, under unbalanced traffic pattern and Poisson arrivals; the parameter ξ represents the percentage of traffic sent from each input module to the output module numbered with the same index.

A very good agreement is obtained between analytical (solid lines) and simulation results (points) in all the presented scenarios.

Acknowledgment

The authors wish to thank Prof. M. A. Marsan and Prof. F. Neri for their helpful suggestions.

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