Embedding Unequal Error Protection into Turbo Codes
Marco Grangetto    Enrico Magli    Gabriella Olmo
Dipartimento di Elettronica - Politecnico di Torino
Corso Duca degli Abruzzi 24 - 10129 Torino - Italy
grangetto(magli,olmo)@polito.it

Abstract

In this paper a technique to embody Unequal Error Protection into turbo codes is presented. Through proper optimization of the turbo codes interleaver different levels of error protection can be accommodated within the same encoded block. The effectiveness of the proposed approach is demonstrated in the case of transmission of JPEG2000 coded images across AWGN channel. The Unequal Error Protection offered by the turbo code is able to well match the progressive nature of the encoded image, so improving the quality of the decoded images at the receiver.

1 Introduction

In recent times, the growing demand for wireless communications has led to an intense effort towards the development of new physical layer solutions. It is a widely agreed opinion [1] that the future of research and development in the field of multimedia lies in the wireless applications. This is also substantiated by the work of the ISO/ITU-T standardization committees, which have foreseen in the recent image and video compression standards (JPEG2000, MPEG-4, H.263++) specific error resilience capabilities in order to achieve robustness in an error prone environment. Several strategies have been devised for the design of robust codecs [2]; in this paper we focus on image coders, but it is worth pointing out that the proposed technique can be applied to any quality or priority scalable source. In particular we consider the JPEG2000 image compression standard [11], currently in advanced publication stage.

Plenty of error resilience techniques for image coding have been proposed in the literature: an excellent survey is given in [2]. However, even though the mere bitstream resilience eases error detection and concealment, reliable wireless multimedia communication can be achieved only with a joint design of the whole transmission chain, so as to achieve codestream robustness through some form of unequal error protection at the application level [3]. In [3] the advantage of joint source-channel approach is demonstrated by optimal allocation of Reed-Solomon FEC codes in the case of image transmission through lossy packet network.

It is well known that the third generation of mobile communications [4] will offer different services, ranging from voice transmission to high-rate packet data transmission. The latter service will work at very low bit error rate (BER) and therefore requires powerful channel coding. Parallel concatenated convolutional codes (PCC’s) or turbo codes [5, 6, 7] are known to exhibit performance close to the Shannon capacity limit. In this paper we demonstrate the possibility of embedding Unequal Error Protection capabilities within PCC’s through proper interleaver design. This approach can lead to the extension of the joint source-channel coding, traditionally applied by means of variable rate block codes [3] or convolutional codes, to the case of turbo codes.

The possibility of achieving Unequal Error Protection (UEP) with turbo codes was investigated in previous works [8, 9]. In [8] UEP is obtained by puncturing low rate codes, employing different interleavers for each protection level. The authors in [9] propose a single generalized circular shift interleaver capable of accomplishing UEP through code puncturing. In this paper we introduce a new interleaver strategy yielding UEP even in absence of code puncturing. The proposed technique is well suited for transmission of fully progressive coded stream, i.e. images coded by popular SPIHT [10] or the novel JPEG 2000 [11] codecs.

This paper is organized as follows. In Sect. 2 the proposed approach is described in details. In Sect. 3 experimental results are presented in the case of transmission across the AWGN channel. Finally in Sect. 4 conclusions are drawn.

2 Interleaver optimization

Let us consider a progressive encoded stream, i.e. most important information is encoded and made
available at the decoder at beginning of the compressed stream. In the case of the JPEG2000 image compression standard [11], this objective is obtained encoding the image in the wavelet domain by means of the powerful Embedded Block Coding with Optimized Truncation (EBCOT) algorithm. It is intuitive that, in such a case, higher quality at the receiver can be guaranteed by better protecting the most important information, generally placed at the beginning of the encoded stream, resorting to a proper unequal protection strategy.

In this work the unequal protection is obtained through proper design of the turbo code interleaver. We consider the PCCC distance spectrum as a function of the position of a bit in the interleaver and we demonstrate that it is possible to adjust the code free distance according to the position in the data block. Let us assume that the all-zeros data word is transmitted and that there is a single dominating error event $e$. Invoking the usual union bound argument we can obtain the following simple approximation for the BER in the position $i$:

$$P_b[i] = \frac{w_e}{2N} \text{erfc} \left( \sqrt{\frac{r_\epsilon d_e[i] E_b}{N_0}} \right)$$

(1)

where $N$ represents the interleaver length, $r_\epsilon$ the code rate, $w_e$ the weight of the error data word, and $d_e[i]$ the weight of the error event produced by word $e$ shifted in position $i$ within the interleaving block of size $N$.

The technique proposed in this paper is based on the idea that the codeword weight $d_e = d_e[i]$ can be adapted according to the source bit priority, as a function of the position $i$ within the interleaved data block of size $N$. This objective is accomplished through the design of “position selective” (PS) interleaver, able to match the rate-distortion function of the input bit-stream.

Let us introduce the permutation vector $\Pi$; the notation $\Pi(i) = j$ means that the $j$-th input bit is moved to the $i$-th position at the output. Given an input binary sequence $c$, in the following we will refer to the scrambled sequence as $\Pi(c)$. The PS interleaver is based on the simple iterative construction presented in Fig. 1. The interleaver positions are progressively selected in order to maximize the interleaving distance: the first two positions ($j = 0, 1$) are placed on interleaver tail and head respectively. Then the interleaver grows up in a dyadic way filling the median positions between those assigned in the previous steps. In Fig. 1 the construction is stopped when the interleaver dimension is equal to 9.

![Figure 1: Recursive interleaver construction](image)

Table 1: Convolutional encoders output sequences and corresponding Hamming weight $W_H$ when the error pattern is respectively placed at the beginning ($c_0$) and at the end ($c_1$) of the input sequence

<table>
<thead>
<tr>
<th>Input seq.</th>
<th>Parity check</th>
<th>$W_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>110100000</td>
<td>101110000</td>
</tr>
<tr>
<td>$\Pi(c_0)$</td>
<td>101000000</td>
<td>111011111</td>
</tr>
<tr>
<td>$c_1$</td>
<td>000011001</td>
<td>000010101</td>
</tr>
<tr>
<td>$\Pi(c_1)$</td>
<td>010001100</td>
<td>111010010</td>
</tr>
</tbody>
</table>

The simple interleaver proposed in Fig. 1 exhibits a selective behavior according to the position one is considering. Let us employ a rate $r_e = 1/3$ parallel concatenated code obtained with two recursive systematic convolutional codes with code generators in octal form $(g_1, g_2) = (31, 27)$ and the simple progressive interleaver with $N = 9$ obtained in Fig. 1. The sequence $e = 11001$ is a minimum weight terminating error pattern for the component codes. In Tab. 1 we compare the Hamming weight $W_H$ of the convolutional encoders outputs when the error pattern is respectively placed at the beginning ($c_0 = 110010000$) and at the end ($c_1 = 000011001$) of the input sequence. It follows that, when the error event is placed in position $i = 0$ in the interleaver, the overall weight of the PCCC encoded sequence is $d_e[0] = 7 + 11 - 3 = 15$. On the other hand the weight of the encoded sequence when $i = 4$, which corresponds to the second case in Tab. 1, is $d_e[4] = 7 + 8 - 3 = 12$. It is evident that, according to Eq. 1, the permutation strategy is more effective in the first case, yielding higher protection to errors when they appear at the beginning of the interleaving frame.

The proposed PS interleaver exhibits unequal error protection capability but further work is needed in order to obtain good average performance for practical interleaver dimensions. In the case of long interleavers it is clearly useless to apply the recursive construction presented in Fig. 1 along all available positions. The proposed design rule would provide the first positions with large interleaving distances but, on the other hand, the last bits would be placed in almost ad-
jacent cells. The solution to this problem is simple yet effective: we employ a PS kernel of reduced dimension to scramble a subset of all available positions. In the most simple case the same PS permutation could be applied to even and odd positions of \( \Pi \) respectively, increasing by a factor 2 the lower interleaving distance. This procedure can be straightforwardly extended to the \( M \)-ary case in order to impose larger lower bounds on the interleaving distance. Moreover the proposed strategy presents an high degree of modularity that could simplify fast interleaver implementations. In the following the notation \( PS(N, M) \) will be used to identify the interleaver of length \( NM \) obtained with \( M \) PS kernels of length \( N \).

3 Experimental results

The proposed PS interleaver family was tested in the case of transmissions across AWGN channel and performances were compared with respect to random interleaver with the same length. Values of \( E_b/N_0 \) in the range between 0 and 0.5 dB were considered and interleaver lengths of 4 kbit were selected. For our purpose the random interleaver represents a significant term of comparison since for low values of \( E_b/N_0 \), as in the case of the reported simulations, all interleavers asymptotically exhibit the same performance [7]. The reported results were obtained with a \( r_c = 1/3 \) PCCC, comprising two equal 8 states recursive systematic convolutional codes with code generators in octal form \( (g_1, g_2) = (31, 27) \).

The performance of the proposed interleavers were evaluated in terms of the BER estimated for each position along the interleaving frame. In Fig. 2 the estimated BER on AWGN channel for the PS(513,8) and random interleaver respectively are reported. In the small graph the BER as a function of the position is reported. The random interleaver is characterized by an almost equal behaviour in term of BER along the frame positions \( i \). On the other hand the proposed PS(513,8) interleaver exhibits unequal error protection capability; in the reported graph the 8 PS kernels can be noticed. In the zoomed graph is reported as well the average BER performance on the 513 positions of the PS kernel, compared with the average BER obtained by the random interleaver. It is worth noticing that the PS interleaver, besides achieving unequal error control, does not show impairment with respect to its random counterpart.

In Tab. 2 the simulation results in the case of JPEG2000 image transmission across AWGN channel are reported. These results were obtained with standard 256×256 Lena image, compressed at 0.25 bpp by means of the JPEG2000 standard; PSNR (Peak Signal to Noise Ratio) scalable encoding was selected, employing a large number of quality layers; this choice allow us to test the proposed interleaver in the case of the transmission of a progressive stream; moreover JPEG2000 error resilience tools were enabled in order to provide the image decoder with error detection capability, preventing from critical error propagation during the decompression process. As for the transmission strategy, the JPEG2000 stream was simply chunked filling up four 4 kbit interleaver frames. Within each frame the source bits were placed progressively in less protected positions in order to well match the scalable nature of the stream. In Tab. 2 the performances obtained with 1000 transmissions of the same image are evaluated for different values of \( E_b/N_0 \) in terms of the average \( \mu \) and the standard deviation \( \sigma \) of the decoded PSNR at the receiver. The percentage of decoding failures and the average BER along all interleaver positions are reported as well. The performance of random and PS(257,16) interleavers are compared, pointing out that the PS interleaver is able to produce significant improvements in term of the decoded image quality. In the simulated range of \( E_b/N_0 \) the PS interleaver exhibits a gain between 0.5 and 1 dB in term of the average decoded PSNR. There are significant reductions in the PSNR standard deviation (1 dB or more) as well, suggesting that the delivered image quality in the PS case presents more graceful degradation. Moreover the PS strategy limits the number of decoding failure (fourth column in Tab. 2), since it guarantees a higher degree of protection to the initial part of the JPEG2000 stream which conveys critical side information for the decoding process.

Finally in Fig. 3 the cumulative probability function \( Prob(PSNR < x) \) is reported for PS(257,16) and random (dash line) interleaver in the case \( E_b/N_0 = 0.15 \) dB. This function allows to compare the performance of the two approaches along all PSNR values, giving a more complete vision of the decoded image quality. The performance improvement of the PS interleaver is evident along all values of decoded PSNR. Moreover, the sharp vertical steps, corresponding to a complete frame decoding, are more visible in the random case. On the contrary the PS interleaver presents a more graceful degradation within each frame as demonstrated by the smoother transitions of the corresponding cumulative function.

4 Conclusions

In this paper we have proposed a novel strategy for embedding UEP into turbo codes, exclusively based on proper design of the interleaving rule. We introduced a family of position selective interleavers that exhibits
Table 2: Image transmission results for different values of $E_b/N_0$ in terms of the average $\mu$ and the standard deviation $\sigma$ of the decoded PSNR at the receiver, percentage of decoding failure and average BER.

<table>
<thead>
<tr>
<th>$E_b/N_0$ [dB]</th>
<th>$\mu$ [dB]</th>
<th>$\sigma$ [dB]</th>
<th>failures [%]</th>
<th>BER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>22.12</td>
<td>8.53</td>
<td>19.1</td>
<td>$2 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.20</td>
<td>25.02</td>
<td>6.27</td>
<td>8.3</td>
<td>$9 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.25</td>
<td>26.14</td>
<td>5.15</td>
<td>5.3</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>$PS(257, 16)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>23.60</td>
<td>7.49</td>
<td>12.8</td>
<td>$1 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.20</td>
<td>25.55</td>
<td>5.44</td>
<td>5.7</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.25</td>
<td>26.65</td>
<td>3.68</td>
<td>2.4</td>
<td>$2 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Figure 2: Small graph: estimated BER as a function of the position in the case $E_b/N_0 = 0.25$ dB for PS(513,8) and random interleaver of the same length. Zoomed graph: average BER of the PS kernel.

Figure 3: Cumulative distribution of decoded PSNR for the $256 \times 256$ image Lena in the case $E_b/N_0 = 0.15$ dB for PS(257,16) and random interleaver (dash)
UEP capabilities without the need of code puncturing. The proposed technique is well suited for progressive stream transmission and it was tested in the case of transmission across the AWGN channel of JPEG2000 compressed images. The PS interleaver outperforms random interleaver of equal length in terms of decoded image quality and provides more graceful degradation in poor channel conditions. Ongoing research in this field by the authors is directed towards random PS interleavers and their performance evaluation in presence of fading.

References
[4] ETSI TS 125 212 v4.0.0, “Universal Mobile Telecommunication System (UMTS); Multiplexing and channel coding (FDD)